



National  
Qualifications

**X847/76/11**

**Mathematics  
Paper 1 (Non-calculator)**



Duration — 1 hour 15 minutes

**Total marks — 55**

**SECTION 1 — 44 marks**

Attempt ALL questions.

**SECTION 2 — 11 marks**

Attempt EITHER Part A OR Part B.

**You must NOT use a calculator.**

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

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\* X 8 4 7 7 6 1 1 \*

## FORMULAE LIST

### Circle

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Scalar product

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

### Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

### Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

## SECTION 1 — 44 marks

Attempt ALL questions

1. Find the value of  $k$  for which the equation  $kx^2 + 3x - 4 = 0$  has equal roots. 3

2. Given that  $f(x) = (x^2 + 1)^5$ , find  $f'(1)$ . 3

3. A function  $f(x)$  is defined on  $\mathbb{R}$ , by

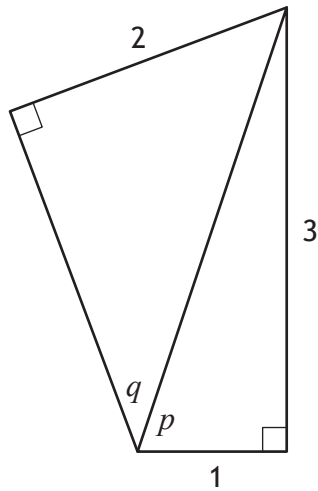
$$f(x) = \frac{x+3}{2}.$$

Find the inverse function,  $f^{-1}(x)$ . 3

4. Determine whether the line passing through  $(-4, 2)$  and  $(2, -7)$  is perpendicular to the line with equation  $3y = 2x + 9$ . 3

[Turn over

5. Two right-angled triangles are shown below.



(a) Determine the value of

(i)  $\sin p$

1

(ii)  $\cos q$ .

2

(b) Find the exact value of  $\cos(p + q)$ .

3

6. Functions  $f$  and  $g$  are defined on  $\mathbb{R}$  by

- $f(x) = 2x + 5$

- $g(x) = x^2 - 2x$ .

(a) Find an expression for  $f(g(x))$ .

2

(b) Find an expression for  $g(f(x))$ .

1

(c) Express  $g(f(x)) - f(g(x))$  in the form  $a(x+b)^2 + c$ .

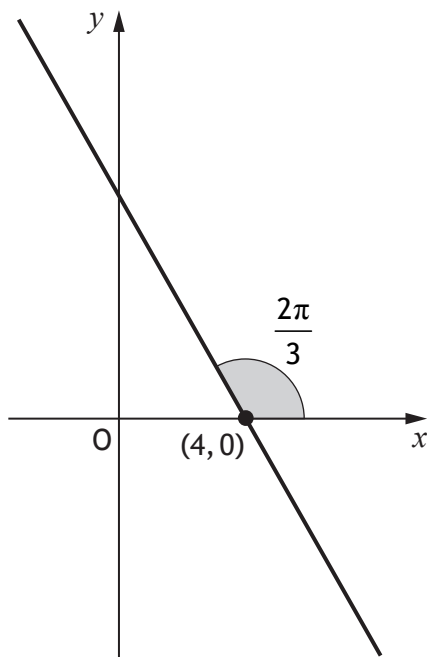
4

7. Find  $\int 6 \cos\left(3x + \frac{\pi}{4}\right) dx$ .

2

8. A line makes an angle of  $\frac{2\pi}{3}$  with the positive direction of the  $x$ -axis.

It passes through the point  $(4, 0)$ .



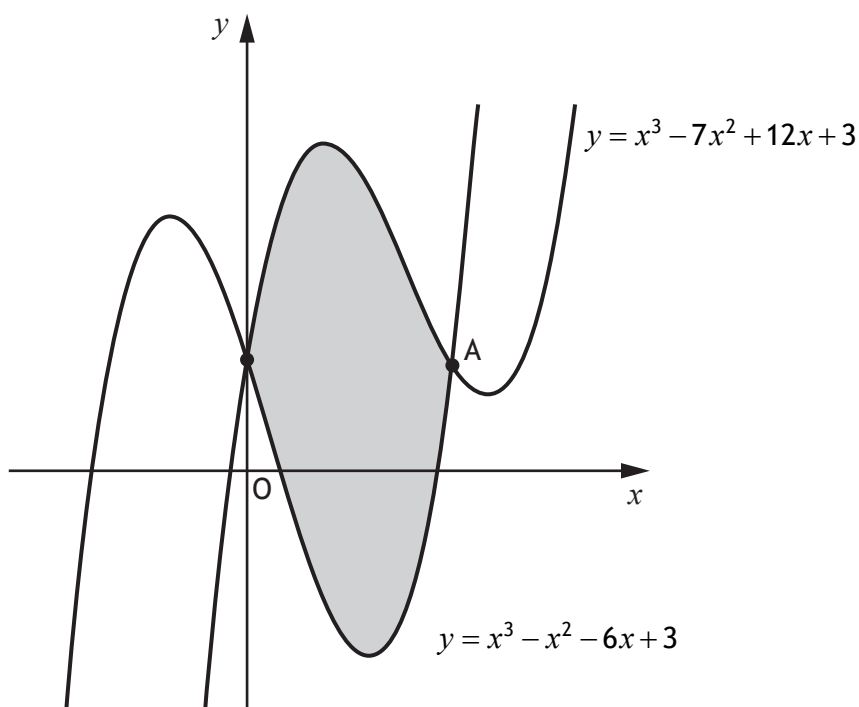
Determine the equation of the line.

3

[Turn over

9. The diagram shows the curves with equations  $y = x^3 - 7x^2 + 12x + 3$  and  $y = x^3 - x^2 - 6x + 3$ .

The curves intersect on the  $y$ -axis and at point A.



- (a) Find the  $x$ -coordinate of A. 2
- (b) Calculate the shaded area. 5
10. Factorise  $6x^3 - 13x^2 + 4$  fully. 4

11. A function,  $f$ , defined on  $\mathbb{R}$ , is such that

- the maximum value of  $f$  is 8
- the maximum occurs when  $x = 6$ .

The function  $g$  is given by  $g(x) = 2f(x) - 9$ .

(a) State the maximum value of  $g$ . 1

The function  $h$  is given by  $h(x) = f(x - 4) + 5$ .

(b) (i) State the maximum value of  $h$ . 1

(ii) State the value of  $x$  when the maximum value of  $h$  occurs. 1

[END OF SECTION 1]

[Turn over

SECTION 2 — 11 marks  
Attempt EITHER Part A OR Part B

## Part A

12. Points A, B, and C are collinear, with B dividing AC.

- A has coordinates (4, 2, -5)
- B has coordinates (7, -4, 1)
- $|\vec{BC}| = 6$

- (a) (i) Find  $|\vec{AB}|$ . 2
- (ii) State the ratio in which B divides AC. 1
- (b) Determine the coordinates of C. 1

13. A sequence is generated by the recurrence relation  $u_{n+1} = \frac{2}{3}u_n + 8$ ,  $u_7 = 20$ .

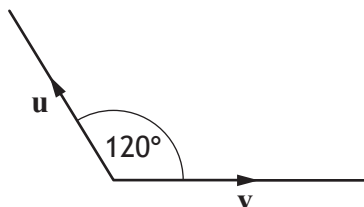
- (a) Determine the value of  $u_5$ . 2

This sequence approaches a limit as  $n \rightarrow \infty$ .

- (b) Determine the limit of this sequence. 2

14. The angle between vectors  $\mathbf{u}$  and  $\mathbf{v}$  is  $120^\circ$ .

$$|\mathbf{u}| = 4 \text{ and } |\mathbf{v}| = 5.$$



Calculate  $\mathbf{u} \cdot (\mathbf{u} + \mathbf{v})$ .

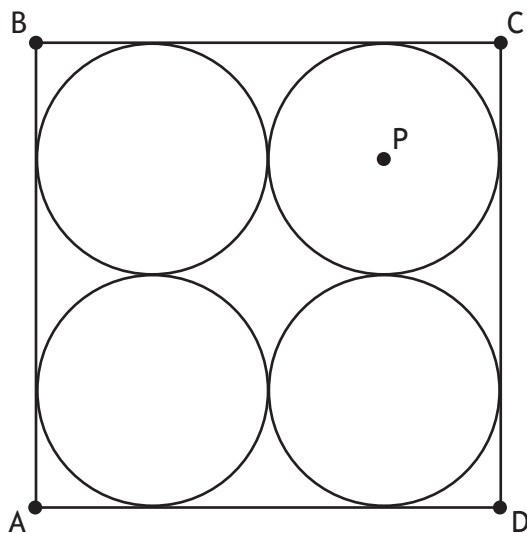
3



## Part B

15. ABCD is a square containing four congruent circles.

A is the point (2, 1), and D is the point (10, 1).



Determine the equation of the circle with centre P.

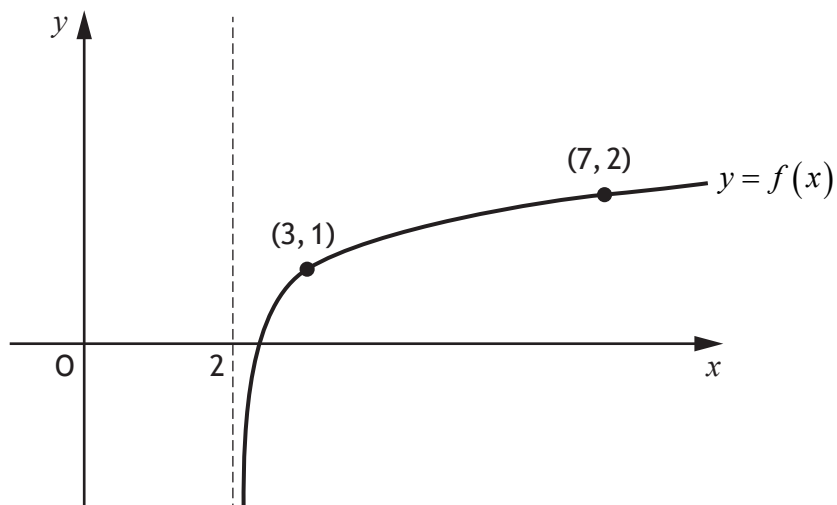
3

16. Evaluate  $\log_2 6 + \log_2 12 - 2\log_2 3$ .

4

[Turn over

17. A logarithmic function,  $f$ , is defined for  $x > 2$ .  
The diagram shows the graph of  $y = f(x)$ .



The inverse function,  $f^{-1}(x)$ , exists.

- (a) On the diagram in your answer booklet, sketch the graph of the inverse function. 2
- (b) Given that  $f(x) = \log_5(x-2) + 1$ , find the coordinates of the point where the graph of  $f^{-1}(x)$  crosses the  $y$ -axis. 2

[END OF SECTION 2]

[END OF QUESTION PAPER]

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National  
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Mark

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**X847/76/01**

**Mathematics Paper 1 (Non-calculator)  
Answer booklet**



Duration — 1 hour 15 minutes



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Fill in these boxes and read what is printed below.

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Town

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Forename(s)

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Surname

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Number of seat

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Date of birth

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Month

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Year

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Scottish candidate number

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**You must NOT use a calculator.**

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QUESTION  
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SECTION 1

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**5.(a)**  
**(i)**

**5.(a)**  
**(ii)**

**5.(b)**



\* X 8 4 7 7 6 0 1 0 4 \*



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6.(a)

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6.(b)

6.(c)



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9.(a)

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9.(b)



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11.(a)

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11.(b)  
(i)

11.(b)  
(ii)

[END OF SECTION 1]



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SECTION 2 — Part A

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12.(a)  
(i)

12.(a)  
(ii)

12.(b)



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13.(a)

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13.(b)



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QUESTION  
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[END OF PART A]



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SECTION 2 — Part B

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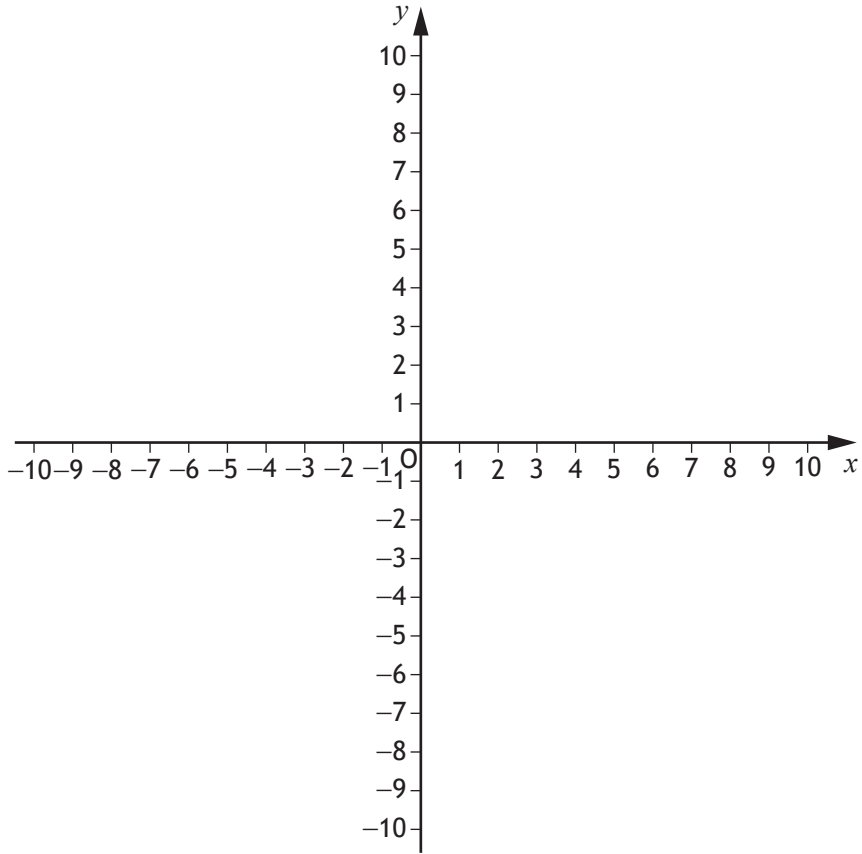
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An additional diagram, if required, can be found on *page 15*.

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17.(a)



17.(b)

[END OF SECTION 2]



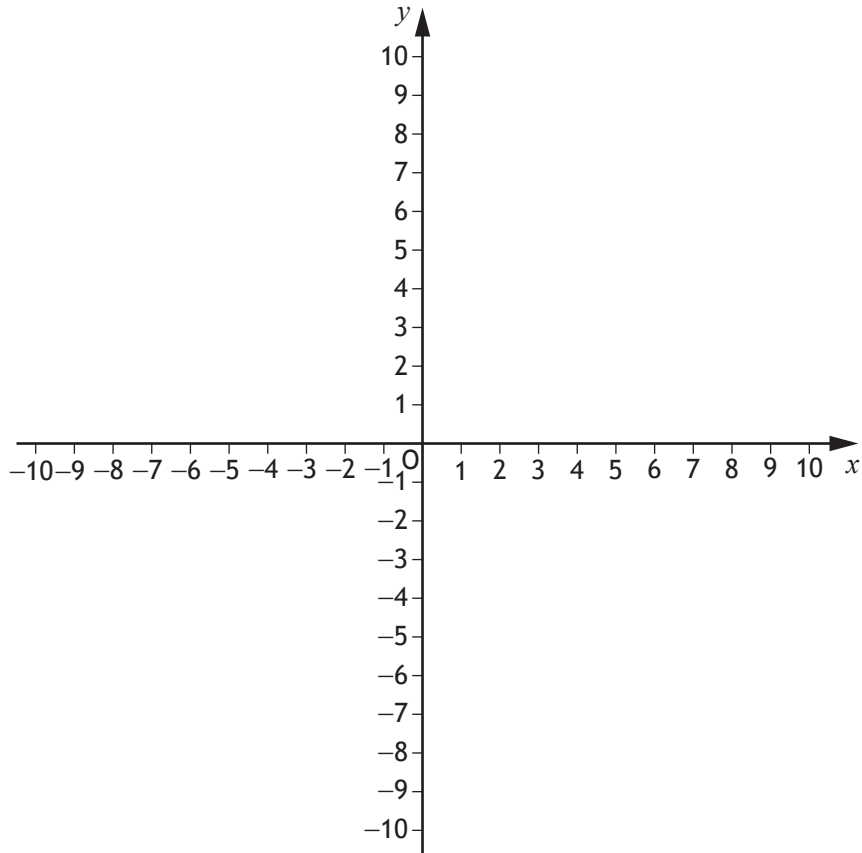
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QUESTION  
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### ADDITIONAL SPACE FOR ANSWERS

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Additional diagram for question 17(a).



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QUESTION  
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ADDITIONAL SPACE FOR ANSWERS

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QUESTION  
NUMBER

## ADDITIONAL SPACE FOR ANSWERS

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Question no	Marks/Grades	



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National  
Qualifications

**X847/76/12**

**Mathematics  
Paper 2**

Duration — 1 hour 30 minutes

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**Total marks — 65**

**SECTION 1 — 52 marks**

Attempt ALL questions.

**SECTION 2 — 13 marks**

Attempt EITHER Part A OR Part B.

**You may use a calculator.**

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\* X 8 4 7 7 6 1 2 \*

## FORMULAE LIST

### Circle

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre  $(-g, -f)$  and radius  $\sqrt{g^2 + f^2 - c}$ .

The equation  $(x - a)^2 + (y - b)^2 = r^2$  represents a circle centre  $(a, b)$  and radius  $r$ .

### Scalar product

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$ , where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$

or  $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$  where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

### Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

### Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

### Table of standard integrals

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$

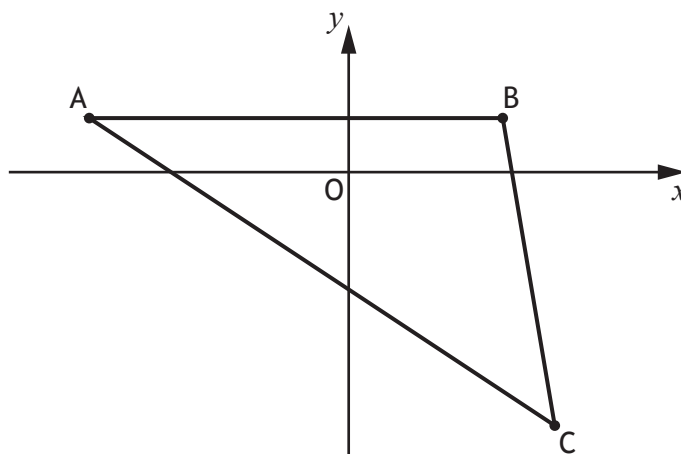
## SECTION 1 — 52 marks

Attempt ALL questions

1. Determine the equation of the tangent to the curve  $y = 2x^3 - 8x^2 + 14$  at the point where  $x = 3$ . 4
2. Find  $\int \frac{6}{(x+5)^{\frac{3}{2}}} dx, x > -5$ . 3
3. Given  $h(t) = \sin\left(2t + \frac{\pi}{6}\right)$ , determine the rate of change of  $h$  when  $t = 10$ . 3

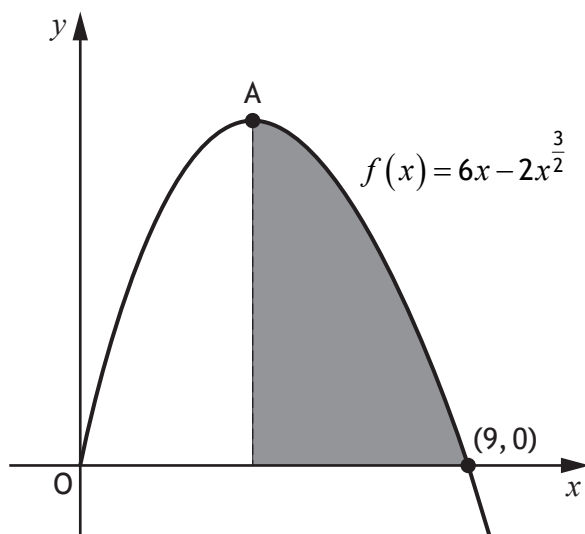
[Turn over

4. Triangle ABC has vertices A  $(-5, 1)$ , B  $(3, 1)$  and C  $(4, -5)$ .



- (a) The line  $L_1$  is the altitude through B.  
Find the equation of  $L_1$ . 3
- (b) The line  $L_2$  is the perpendicular bisector of AB.  
Find the equation of  $L_2$ . 3
- (c) Determine the coordinates of the point of intersection of  $L_1$  and  $L_2$ . 1
5. (a) Express  $3\cos t^\circ + 5\sin t^\circ$  in the form  $k\sin(t+a)^\circ$ ,  $k > 0$ ,  $0 < a < 360$ . 4
- (b) A function,  $f$ , is defined by  $f(t) = 3\cos t^\circ + 5\sin t^\circ$ ,  $0 \leq t < 360$ .
- (i) State the minimum value of  $f(t)$ . 1
- (ii) Determine the value of  $t$  where this minimum occurs. 1

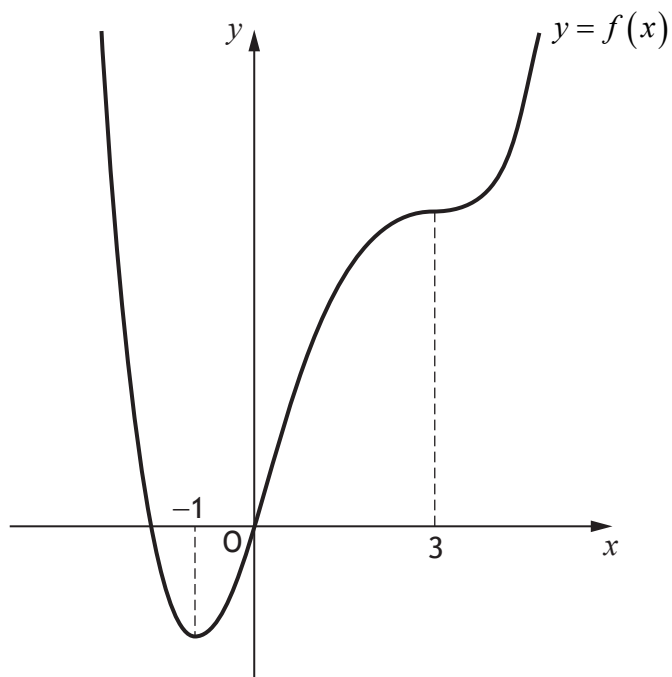
6. The graph of the function  $f(x) = 6x - 2x^{\frac{3}{2}}$ ,  $x \geq 0$  is shown.  
The point A is a stationary point of  $f(x)$ .



- (a) Determine the  $x$ -coordinate of the stationary point A. 3
- (b) Hence calculate the shaded area. 4

[Turn over

7. The diagram shows the graph of  $y = f(x)$ , which has stationary points at  $x = -1$  and  $x = 3$ .



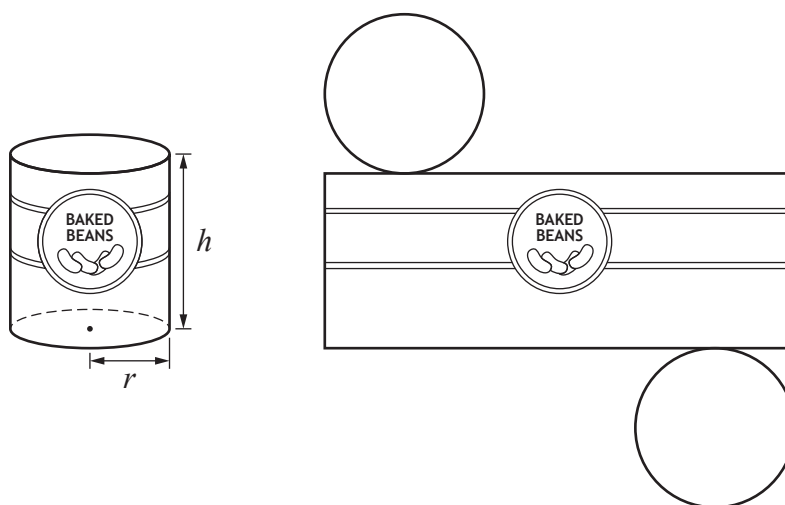
On the diagram in your answer booklet, sketch a possible graph of  $y = f'(x)$ .

3

8. Solve the equation  $2\sin(3x - 60)^\circ + 1 = 0$ ,  $0 \leq x < 180$ .

4

9. A cylindrical tin of baked beans has a volume of  $450 \text{ cm}^3$ .  
The radius of the tin is  $r$  cm and its height is  $h$  cm.  
A net of the tin is shown in the diagram.



- (a) Show that the surface area of the tin,  $A$  square centimetres, is given by

$$A(r) = 2\pi r^2 + \frac{900}{r}. \quad 3$$

- (b) Determine the radius that will minimise the surface area. 6

10. (a) Show that  $2 \tan x \cos^2 x = \sin 2x$ , where  $-\frac{\pi}{2} < x < \frac{\pi}{2}$ . 2

- (b) Given that

- $\frac{dy}{dx} = 6 \tan x \cos^2 x$ , and
- $y = 3$  when  $x = 0$ ,

express  $y$  in terms of  $x$ . 4

[END OF SECTION 1]

[Turn over

SECTION 2 — 13 marks  
Attempt EITHER Part A OR Part B

## Part A

11. (a) Given  $A(3, 1, 8)$ ,  $B(-2, 5, 1)$  and  $C(7, -6, 3)$ ,  
express  $\vec{AB}$  and  $\vec{AC}$  in component form. 2

- (b) Hence calculate the size of angle BAC. 4

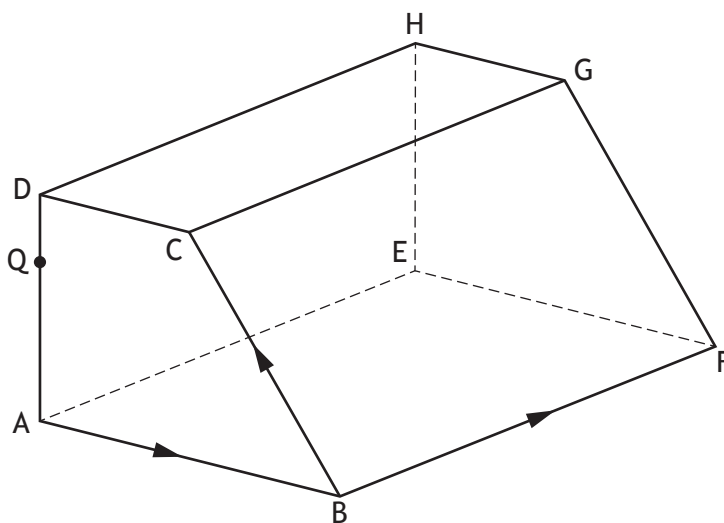
12. A sequence of real numbers is such that
- the terms of the sequence satisfy the recurrence relation  
 $u_{n+1} = 9u_n - 440$
  - $u_{n+1} > u_n$  for all values of  $n$ .

The difference between two particular terms,  $u_{k+1}$  and  $u_k$ , is 1000.

Determine, algebraically, the value of  $u_k$ . 3



13. ABCD,EFGH is a prism.



- $\vec{AB} = \begin{pmatrix} 8 \\ -4 \\ 6 \end{pmatrix}$ ,  $\vec{BC} = \begin{pmatrix} -7 \\ 5 \\ 3 \end{pmatrix}$  and  $\vec{BF} = \begin{pmatrix} 7 \\ 11 \\ -2 \end{pmatrix}$ .

- $\vec{AB} = 2\vec{DC}$ .

(a) Express  $\vec{CF}$  in component form.

1

(b) Hence, or otherwise, express  $\vec{DF}$  in component form.

1

(c) The point Q lies on the line AD.

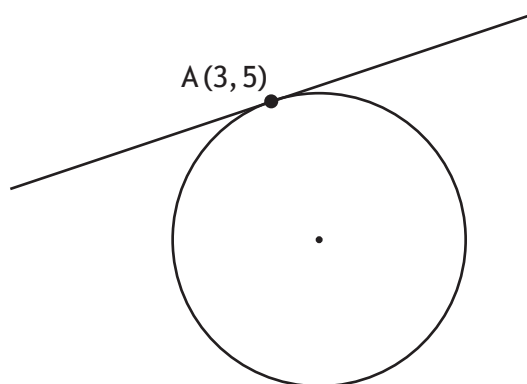
Given that  $\vec{QF} = \begin{pmatrix} 17 \\ 5 \\ 0 \end{pmatrix}$ , find  $\vec{QD}$ .

2

[Turn over

## Part B

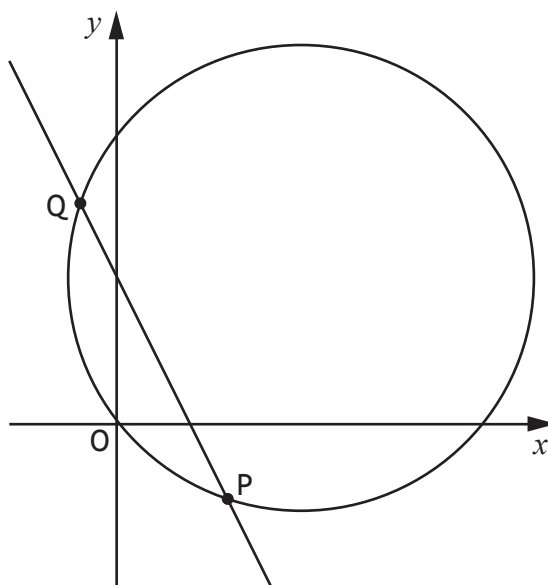
14. The point  $A(3, 5)$  lies on the circle with equation  $x^2 + y^2 - 10x + 2y - 14 = 0$ .



Find the equation of the tangent to the circle at A.

4

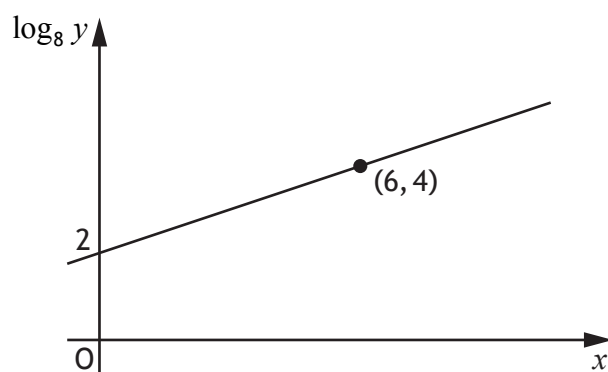
15. The line  $y = 4 - 2x$  intersects the circle  $x^2 + y^2 - 10x - 8y + 1 = 0$  at the points P and Q.



Find the coordinates of the points of intersection.

4

16. Two variables,  $x$  and  $y$ , are connected by the equation  $y = ab^x$ .  
The graph of  $\log_8 y$  against  $x$  is a straight line as shown.



Find the values of  $a$  and  $b$ .

5

[END OF SECTION 2]

[END OF QUESTION PAPER]

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**X847/76/02**

**Mathematics Paper 2  
Answer booklet**

Duration — 1 hour 30 minutes



Fill in these boxes and read what is printed below.

Full name of centre

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Town

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Forename(s)

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Surname

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Number of seat

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Date of birth

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Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

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QUESTION  
NUMBER

SECTION 1

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1.

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QUESTION  
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3.

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4.(a)



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QUESTION  
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4.(b)

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4.(c)



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QUESTION  
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5.(a)

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5.(b)  
(i)

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(ii)



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6.(a)

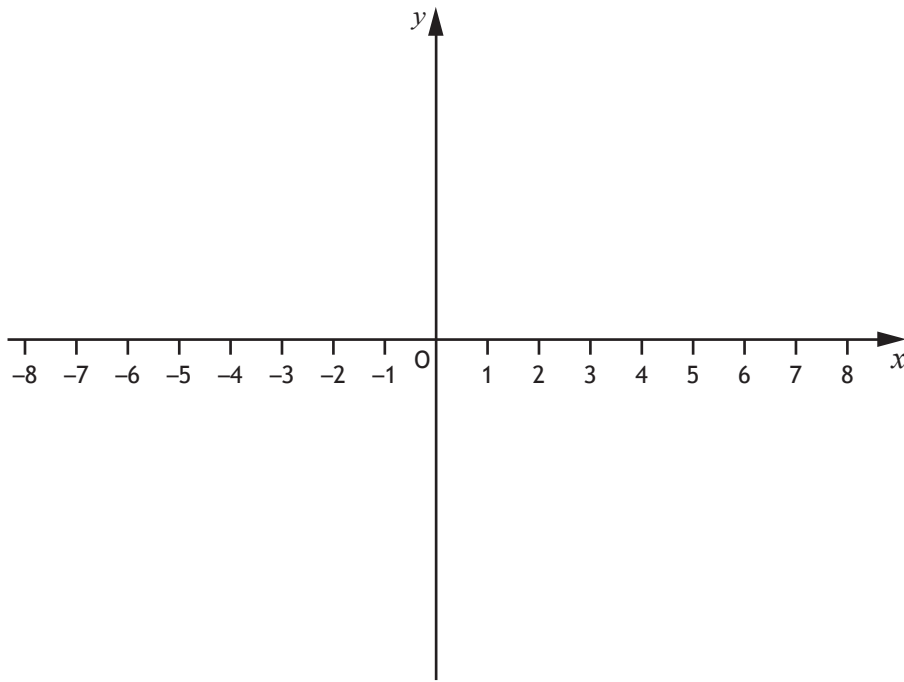
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6.(b)



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7.



An additional diagram, if required, can be found on *page 18*.



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QUESTION  
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9.(a)

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9.(b)

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10.(a)

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10.(b)

[END OF SECTION 1]



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SECTION 2 — Part A

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11.(a)

11.(b)



\* X 8 4 7 7 6 0 2 1 2 \*



QUESTION  
NUMBER

12.

DO NOT  
WRITE IN  
THIS  
MARGIN



\* X 8 4 7 7 6 0 2 1 3 \*

QUESTION  
NUMBER

13.(a)

DO NOT  
WRITE IN  
THIS  
MARGIN

13.(b)

13.(c)

[END OF PART A]



\* X 8 4 7 7 6 0 2 1 4 \*

QUESTION  
NUMBER

14.

SECTION 2 — Part B

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WRITE IN  
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MARGIN



\* X 8 4 7 7 6 0 2 1 5 \*

QUESTION  
NUMBER

15.

DO NOT  
WRITE IN  
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MARGIN



\* X 8 4 7 7 6 0 2 1 6 \*

QUESTION  
NUMBER

16.

DO NOT  
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[END OF SECTION 2]



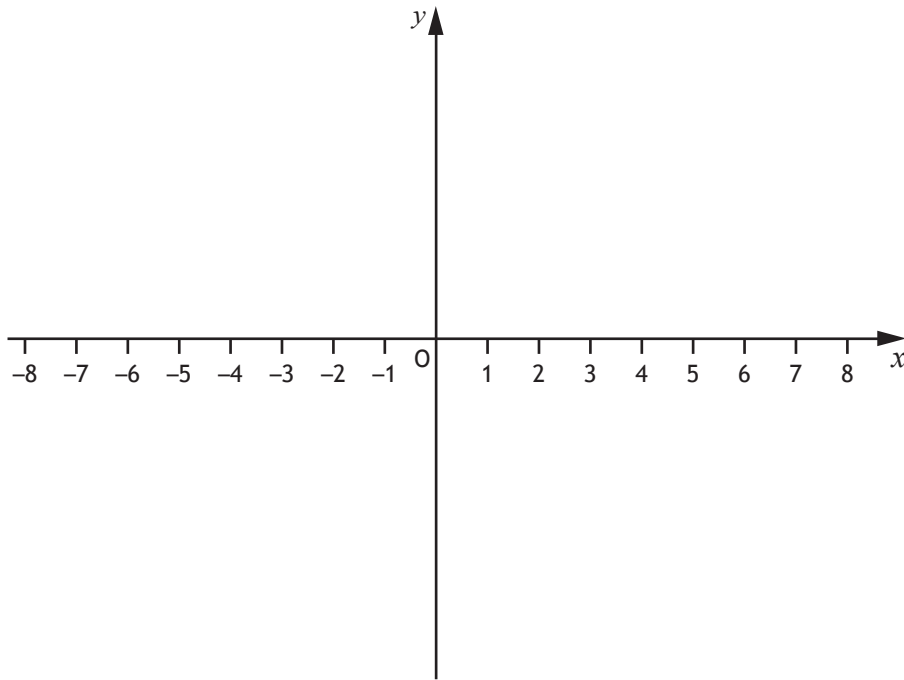
\* X 8 4 7 7 6 0 2 1 7 \*

QUESTION  
NUMBER

### ADDITIONAL SPACE FOR ANSWERS

DO NOT  
WRITE IN  
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MARGIN

Additional diagram for question 7.





QUESTION  
NUMBER

## ADDITIONAL SPACE FOR ANSWERS

DO NOT  
WRITE IN  
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\* X 8 4 7 7 6 0 2 1 9 \*



QUESTION  
NUMBER

## ADDITIONAL SPACE FOR ANSWERS

DO NOT  
WRITE IN  
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\* X 8 4 7 7 6 0 2 2 0 \*



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\* X 8 4 7 7 6 0 2 2 1 \*

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\* X 8 4 7 7 6 0 2 2 2 \*

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\* X 8 4 7 7 6 0 2 2 3 \*

